Let's Get Beyond "Covering It" and Go for Depth

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for

NCTM Regional Conference
Cleveland, OH
Thursday, 16 October 2008
Session #118, Grand Ballroom, Crowne Plaza
3:30 - 4:30 p.m.

Web Site: http://bgmath.org
(click on **Brahier**)

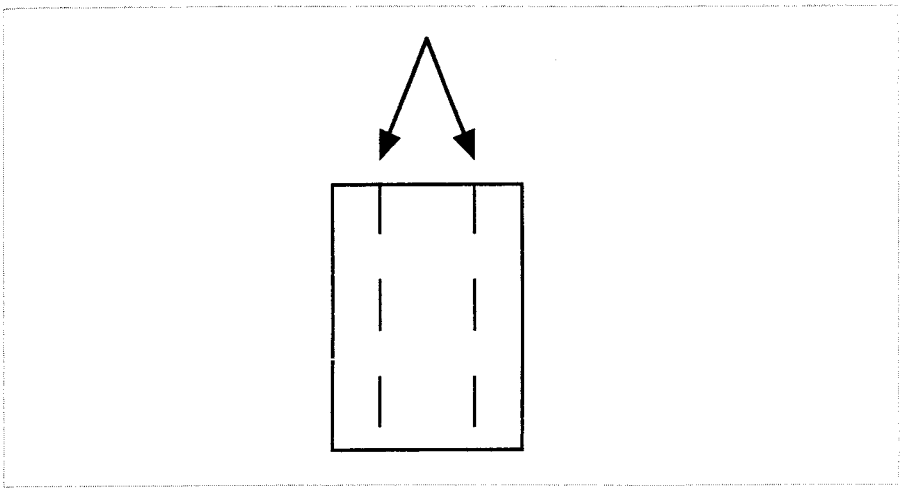

Figure 2–1 Folding the paper to make the gutter

Description of the Activity

The Rain Gutter Problem

Begin the activity by distributing one piece of 8-inch by 10-inch card stock paper to each student. Explain to the class that they are in the business of making rain gutters and that their job is to bend the aluminum twice to make a three-dimensional gutter out of the piece of "aluminum," as shown in Figure 2–1.

If the folds are the same distance from each edge, and the paper is turned sideways, one can see a cross-sectional view of the rain gutter. Students will recognize that the cross section is a rectangle (without a "top"). Ask the students how the dimensions of the rectangle will affect the flow of water along the gutter. They should recognize that as the area of the rectangle is increased, the capacity of the gutter to transport water is also increased. This discussion, then, sets the students up to explore the key question: *What is the maximum possible area of the rectangle that makes up the cross section, given the restriction that the width of the aluminum is 8 inches?*

Students should be encouraged to use their rulers, make sample folds, and calculate various possible solutions to the area problem. The students are likely to ask if the length of the paper (10 inches) has anything to do with the problem. The teacher can explain how the gutter will have the same cross-sectional shape, regardless of whether the piece of aluminum is 10 inches or 10 feet in length, so the fact that their paper is 10 inches long is irrelevant. Also, students typically inquire about whether the folds have to be integral values, and it is important that they realize that they can fold it at any distance they choose, whether the number of inches is an integer or not.

As they investigate folds of different lengths, students will likely generate examples such as those shown in Figure 2–2.

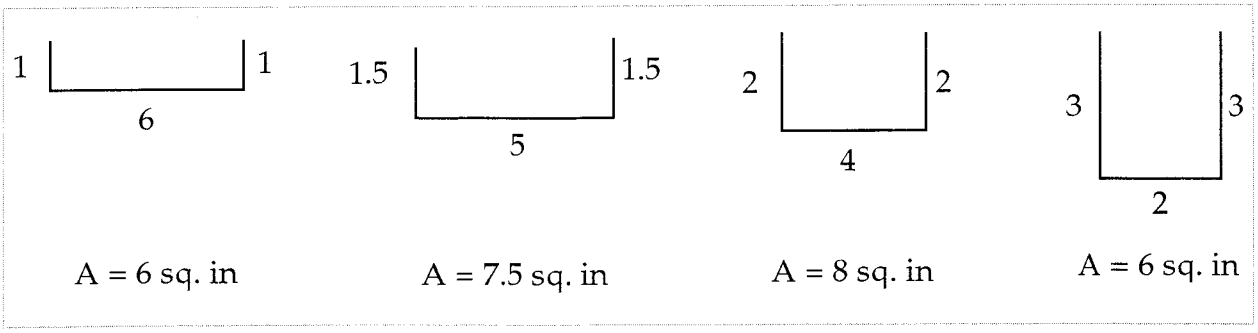

Figure 2–2 Examples of cross-sectional rectangle folds and related areas

Continue to ask students what they have found as their maximum area "so far" and if there appear to be any limits to how high the area can get. They should begin to realize that two different rectangles can make cross sections that have the same area, such as 1 inch by 6 inches and 3 inches by 2 inches, but that they cannot find a rectangle that exceeds 8 square inches as its area. As a result, they will likely conclude that 8 square inches is the solution. When the class reaches this solution, the teacher should press the students to present a convincing argument that their answer is correct.

Using a Table to Represent the Problem

An effective, field-tested way to help students to justify their solution is by creating a simple table. Using half-inch increments, students can generate a table of rectangle dimensions that includes the resulting area for each cross section, such as shown in Figure 2–3.

After creating this table with his class, Mr. Lang leads his class in a discussion to assess his students' understanding of the problem thus far.

Fold (Height)	Base	Area
0	8	0
0.5	7	3.5
1	6	6
1.5	5	7.5
2	4	8
2.5	3	7.5
3	2	6
3.5	1	3.5
4	0	0

Figure 2–3 Dimensions and areas of various rectangles

GEOMETRIC INTERPRETATIONS OF THE EQUATION SOLVING PROCESS

Daniel J. Brahier

Solve each of the following equations. Use **graphing** as the method of solution. So, when we think about $x-2=5$, we will explore the intersection of the following functions:

$$\begin{cases} y_1 = x-2 \\ y_2 = 5 \end{cases}$$

In each case, identify what type of function is represented by each side of the equation and, geometrically, what the solutions will look like.

- $x-2=5$
- $2x+3=x-4$
- $3x+2=3x-7$
- $2(3x-2)=x+3x+8$
- $x^2-6x=-5$
- $2x^2-5x-12=0$
- $2x^2-3x+4=x^2-5x+7$
- $x^2+8x+3=2x+10$

EIGHTHGRADEADVANCEDMATHEMATICS

Function Project #1

Rationale:

The purpose of this project is to get you to apply some of the concepts of functions to a real-life situation of your own.

Timeline:

Thursday, 22 October - Project Assigned
Wednesday, 28 October - Project Paper Due Date
Wednesday & Thursday, 27-28 October - 5-Minute Presentations on Functions

Assessment:

This short project will "count" the same weight as a test score of 100 points, as follows:

70 points -- Written Paper
30 points -- Presentation to Class

Requirements:

Each person is to come up with one, real-life example of a function that affects you or your family. You may want to discuss this with your parents to come up with a good example to use. Then, in a paper, you are to "write up" the following eight items:

A. Explain/Describe the function that you selected and include why you selected it (e.g., "My parents have a voicemail service, so I chose a function that related the monthly cost of the service to the number of messages left on voicemail. I knew that the monthly fee was $8.00 and that each call cost 50¢ ...").
B. Explain which variable is independent, which is dependent and explain how you know.
C. Create a table showing several values that relate the variables.
D. Draw an accurately, fully-labeled graph of the function on an attached piece of graph paper.
E. Write a statement of your relationship in "function language."
F. Find a recursive formula that describes your function.
G. Find a closed formula and an equation (using y and x) that describe your function.
H. Determine whether your function is discrete or continuous and explain how you know.

Then, in class, on your assigned day, you are to present your project to the class and provide us with an overview of your function. You have about 5 minutes to make your presentation, and you will be assessed on your ability to accurately explain and describe the function you studied. Most effective presentations include some kind of visual, so you might want to use the overhead graphing calculator or make a poster to help the class to visualize your function.

Good luck!!!

EIGHTH GRADE ADVANCED MATHEMATICS
FUNCTIONS PROJECT #1

Name: _____

Class Presentation:

	1 (Low)	2	3	4 (High)
Preparation	___	___	___	___
Knowledge of Function	___	___	___	___
Quality of Explanation	___	___	___	___
Use/Quality of Visual Aid	___	___	___	___
Communication Skills (including grammar, etc.)	___	___	___	___

Comments:

SCORE = _____ out of 20 points

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EIGHTH GRADE ADVANCED MATHEMATICS
FUNCTIONS PROJECT #1

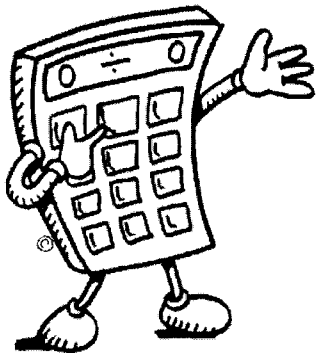
Name: _____

Written Paper:

	1 (Low)	2	3	4 (High)
Required Questions Answered	___	___	___	___
Accuracy of Mathematics	___	___	___	___
Thoroughness of Responses	___	___	___	___
Clarity of Writing	___	___	___	___
Neatness of Written Work	___	___	___	___

Comments:

SCORE = _____ out of 20 points



NCTM Standards for the Teaching and Learning of Mathematics

July 2007

These Standards are "the core dimensions of teaching and learning mathematics" and "represent major arenas of teachers' work that are central to defining what happens in mathematics classrooms."

Assumptions

1. Mathematical processes are central to the curriculum.
2. *What* students learn is fundamentally connected with *how* they learn it.
3. ALL students can learn to think mathematically.
4. Teaching is a complex practice.

Knowledge Standards

1. Knowledge of Mathematics and General Pedagogy
2. Knowledge of Student Mathematical Learning

Implementation Standards

3. Worthwhile Mathematical Tasks
4. Learning Environment
5. Discourse

Analysis Standards

6. Reflection on Student Learning
7. Reflection on Teaching Practice

ORANGE GROVE PROBLEM

A certain farmer in Florida had an orange grove. In his grove were 120 trees. Each tree ordinarily produces 650 oranges. He is interested in raising his orange production and knows that because of lost space and sunlight, every additional tree that he plants will cause a reduction of 5 oranges from each tree.

What is the maximum number of oranges that he will be able to produce in his grove, and how many trees will it take to reach this maximum?

SUMMARY --
3 WAYS TO CREATE RICH LESSONS:

- Select a problem/activity/lesson that lends itself to deep discussion, multiple representations, and connections. (See Brahier's *Panning for Gold* by Heinemann Publishers, from which the Rain Gutter Problem was pulled.)

- Take the problems you would ordinarily solve anyway and tweak them to make them better. For example, instead of just "talking about" Pythagorean Triples, have students investigate patterns with triples and ways to generate additional examples.

- Listen to your students and pick up on their questions for future plans. Questions about advertisements and newspaper articles are natural opportunities for developing lesson ideas.

A GOOD TEACHER

"Now, therefore, the superior teacher leads the students and does not pull them along. The teacher urges them to go forward and does not suppress them. The teacher opens the way to them but does not take them to the place.

Leading without pulling makes the process of learning gentle. Urging without suppressing makes the process of learning easy, and opening the way to the students without taking them to the place makes them think for themselves.

Now, if the process of education is made gentle and easy and if the students are taught to think for themselves, we may call the person a <u>good teacher.</u>"

-- Confucius

You Might Be a Math Teacher if:

10. You come home many days with a horizontal white line across your butt.

9. When you dine out, you can figure out the 15% tip without even thinking.

8. When stating phone numbers, account numbers, or addresses, you always say "zero" instead of "O."

7. You can average any three two digit numbers instantly, without pencil, paper, or calculator.

6. When you see the word "sin" in print, it has absolutely no religious connotations.

5. When at a convention or conference, you spend your spare time in restaurants or bars writing on napkins.

4. You wake up in the middle of the night and say "I got it!" and scare your spouse half to death.

3. You actually believe that someone can row upstream 24 miles in four hours and that lighthouses are always located on straight coastlines.

2. When someone asks your opinion on the Simpson Ruling, you reply, "It definitely works much better than the Trapezoidal Method."

And the number 1 reason you might be a math teacher:

1. Your significant other looks at what you have been writing at the kitchen table all night and says, "You'd think that someone who understands all that would be making a decent salary!"